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Three-body baryonic $\overline{B} \to \Lambda\,\bar{p}\,\pi$ decays and such

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Abstract. We study the decay rates and spectra of $\overline{B} \to \Lambda\,\bar{p}\,\pi$, $\Sigma^0\,\bar{p}\,\pi$, $\Sigma^-\,\bar{n}\,\pi$, $\Xi^0\,\overline{\Sigma}^+\,\pi$, $\Xi^-\,\overline{\Sigma}^0\,\pi$ and $\Xi^-\,\overline{\Lambda}\,\pi$ modes in the factorization approach. The baryon pairs are produced through vector, axial vector, scalar and pseudoscalar operators. Previous predictions, including ours, are an order of magnitude too small compared to experiment. By incorporating QCD counting rules and studying the asymptotic behavior, we find an earlier relation between the pseudoscalar and axial vector form factors to be too restrictive. Instead, the pseudoscalar and scalar form factors are related asymptotically. Following this approach, the measured $\Lambda\,\bar{p}\,\pi^+$ rate ($\sim 4.0 \times 10^{-6}$) and spectrum can be understood, and Λ should be dominantly left-hand polarized, while we expect $\mathcal{B}(\Sigma^0\,\bar{p}\,\pi^+) \simeq 1.6 \times 10^{-6}$. These results and other predictions can be checked soon.

1 Introduction

Several three-body baryonic B decays such as $\bar{B} \to D^*\,p\,\bar{n}$ [1], $p\,\bar{p}\,K$ [2] and $D^*\,p\,\bar{p}$ [3] have emerged recently, even though there is only one single two-body baryonic mode $\bar{B}^0 \to \Lambda_c^+\,\bar{p}$ that is observed so far [4,5].

It has been argued that three-body baryonic modes could be enhanced over two-body ones [6], by reducing the energy release to the baryons via emitting a fast recoil meson. One consequence is enhancement near the baryon pair threshold in three-body modes. In our study of $B^0 \to D^{*-}\,p\,\bar{n}$ [7], *assuming factorization*, we obtained $\sim 60\%$ of the experimental rate from the vector current contribution, and the decay spectrum exhibits such a threshold enhancement. The same threshold enhancement effect was predicted for the charmless $\rho\,p\,\bar{n}$ mode [8], and, interestingly, the newly and first ever observed charmless baryonic mode $B \to p\,\bar{p}\,K$ showed a similar feature [2]. The measured decay rate can be understood to some extent [9] and the spectrum can be reproduced by using the factorization approach and QCD counting rule arguments [10]. Other charmless modes such as $p\,\bar{p}\,\pi$, $\Lambda\,\bar{p}\,\pi$, $\Sigma^0\,\bar{p}\,\pi$ have been studied by the factorization assumption and $\mathcal{B}(\Lambda\,\bar{p}\,\pi^+) = (3\text{–}5) \times 10^{-7}$ and $\mathcal{B}(\Sigma^0\,\bar{p}\,\pi^+) = (0.8\text{–}1.8) \times 10^{-6}$ were predicted [9,10].

Recently, Belle reported [11]

$$\mathcal{B}(\Lambda\,\bar{p}\,\pi^+) = (3.97^{+1.00}_{-0.80} \pm 0.56) \times 10^{-6}, \tag{1}$$

and $\mathcal{B}(\Sigma^0\,\bar{p}\,\pi^+) < 3.8 \times 10^{-6}$ at the 90% confidence level. While the $\Lambda\,\bar{p}\,\pi$ decay spectrum exhibits threshold enhancement as expected, the measured rate turns out to be an order of magnitude higher than predicted [9,10]. Furthermore, previous predictions placed $\mathcal{B}(\Sigma^0\,\bar{p}\,\pi^+)$ considerably above $\mathcal{B}(\Lambda\,\bar{p}\,\pi^+)$. If the factorization approach is not to be abandoned, where could things go wrong?

We had noted that the $\overline{B} \to \Lambda\,\bar{p}\,\pi^+$ mode is sensitive to how one treats the vacuum to $\Lambda\bar{p}$ pseudoscalar matrix element [10] under factorization. The analogous situation for the meson case is known to be enhanced.

In this work, we revisit these two modes, as well as some SU(3) related modes such as $\Sigma^-\,\bar{n}\,\pi$, $\Xi^0\,\overline{\Sigma}^+\,\pi$, $\Xi^-\,\overline{\Sigma}^0\,\pi$ and $\Xi^-\,\overline{\Lambda}\,\pi$. With the help of QCD counting rules and taking into account the asymptotic behavior of the baryonic form factors, we can now account for the observed $\Lambda\,\bar{p}\,\pi$ rate and spectra, where the $\Lambda\bar{p}$ production is dominated by the pseudoscalar density. After improving the situation for the $\Lambda\,\bar{p}\,\pi$ rate, we study the Λ polarization, which is known to be useful for constructing CP- and T-violation observables [6]. We are able to make some predictions as well.

Our formulation is given in the next section, followed by results and discussion.

2 Formalism

Under the factorization assumption, the three-body baryonic B decay amplitude consists of two parts. For one, the baryon pair is *current-produced* in association with a B to meson transition. For the other, the B makes a *transition* to a baryon pair and the recoil meson is current-produced [10]. The $B \to p\,\bar{p}\,K$ mode receives both contributions, but the $\Lambda\,\bar{p}\,\pi^+$ mode, and analogously its SU(3) related modes such as $\Sigma^0\,\bar{p}\,\pi^+$, $\Sigma^-\,\bar{n}\,\pi^+$, $\Xi^0\,\overline{\Sigma}^+\,\pi^+$, $\Xi^-\,\overline{\Sigma}^0\,\pi^+$ and $\Xi^-\,\overline{\Lambda}\,\pi^+$, receive *only* the current-produced contribution. We shall apply the term "current-produced" to scalar and pseudoscalar densities as well.

Take, for example, the $\overline{B}^0 \to \Lambda\,\bar{p}\,\pi^+$ decay. Under factorization, the amplitude is [10]

$$\mathcal{M}(\Lambda\,\bar{p}\,\pi^+) = \frac{G_{\rm F}}{\sqrt{2}} \langle \pi^+ | \bar{u}\gamma^\mu(1-\gamma_5)b | \overline{B}^0 \rangle$$

$$\times \left\{ (V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* a_4) \langle \Lambda \bar{p} \,| \bar{s} \gamma_\mu (1-\gamma_5) u|0\rangle \right.$$
$$\left. +2a_6 V_{tb} V_{ts}^* \, \frac{(p_\Lambda + p_{\bar{p}})_\mu}{m_b - m_u} \langle \Lambda \bar{p} \,| \bar{s}(1+\gamma_5) u|0\rangle \right\}. \quad (2)$$

The baryon pair $\Lambda \bar{p}$ is produced from vacuum through $\bar{s}\,\gamma_\mu\,(\gamma_5) u$ and $\bar{s}\,(\gamma_5) u$ operators, while the $\overline{B}^0$ to π^+ transition is induced by $\bar{u}\gamma^\mu b$ current.

Note that, from isospin symmetry, we have

$$\langle \pi^0 | \bar{u}\gamma^\mu b | B^- \rangle = \langle \pi^+ | \bar{u}\gamma^\mu b | \overline{B}^0 \rangle / \sqrt{2},$$

hence $\mathcal{M}(\Lambda \bar{p} \pi^0) = \mathcal{M}(\Lambda \bar{p} \pi^+)/\sqrt{2}$. For these current-produced modes, we have

$$\mathcal{B}(B^- \to \mathbf{B}\overline{\mathbf{B}}' \pi^0) = \frac{\tau_{B^-}}{2\,\tau_{\overline{B}^0}} \mathcal{B}(\overline{B}^0 \to \mathbf{B}\overline{\mathbf{B}}' \pi^+), \quad (3)$$

where $\tau_{\overline{B}^0,\,B^-}$ are the $\overline{B}^0$ and B^- meson lifetimes, and $\mathbf{B}$ stands for some baryon.

The (axial) vector current-produced matrix elements are decomposed as follows:

$$\langle \mathbf{B}\overline{\mathbf{B}}' | V_\mu | 0 \rangle = \quad (4)$$
$$\bar{u}(p_{\mathbf{B}}) \left\{ (F_1 + F_2)\gamma_\mu + \frac{F_2(t)}{m_{\mathbf{B}} + m_{\mathbf{B}'}} \left(p_{\overline{\mathbf{B}}'} - p_{\mathbf{B}}\right)_\mu \right\} v(p_{\overline{\mathbf{B}}'}) \,,$$
$$\langle \mathbf{B}\overline{\mathbf{B}}' | A_\mu | 0 \rangle = \quad (5)$$
$$\bar{u}(p_{\mathbf{B}}) \left\{ g_{\mathrm{A}}\,(t)\,\gamma_\mu + \frac{h_{\mathrm{A}}\,(t)}{m_{\mathbf{B}} + m_{\mathbf{B}'}} \left(p_{\mathbf{B}} + p_{\overline{\mathbf{B}}'}\right)_\mu \right\} \gamma_5 \, v(p_{\overline{\mathbf{B}}'}) \,,$$

where $F_{1,2}$, g_{A} and h_{A} are the induced vector (Dirac and Pauli), axial and the induced pseudoscalar form factors, respectively, and $t \equiv (p_{\mathbf{B}} + p_{\overline{\mathbf{B}}'})^2 \equiv m^2_{\mathbf{B}\overline{\mathbf{B}}'}$. The scalar and pseudoscalar matrix elements associated with the a_6 term of (2) are expressed as

$$\langle \mathbf{B}\overline{\mathbf{B}}' | S | 0 \rangle = f_{\mathrm{S}}\,(t)\; \bar{u}(p_{\mathbf{B}}) v(p_{\overline{\mathbf{B}}'}) \,, \quad (6)$$
$$\langle \mathbf{B}\overline{\mathbf{B}}' | P | 0 \rangle = g_{\mathrm{P}}\,(t)\; \bar{u}(p_{\mathbf{B}}) \gamma_5 \, v(p_{\overline{\mathbf{B}}'}). \quad (7)$$

It is the $g_{\mathrm{P}}(t)$ form factor that is the focus of our attention, where we offer a refined discussion faced with $B \to \Lambda \bar{p} \pi^+$ data.

The scalar and vector matrix elements can be related by the equation of motion, $\langle \mathbf{B}\overline{\mathbf{B}}' | \partial^\mu V_\mu | 0 \rangle = \mathrm{i}\,(m_q - m_{q'})$ $\langle \mathbf{B}\overline{\mathbf{B}}' | \bar{q}\, q' | 0 \rangle$, giving [9,10]

$$f_{\mathrm{S}}(t) = \frac{m_{\mathbf{B}} - m_{\mathbf{B}'}}{m_q - m_{q'}} \, F_1(t). \quad (8)$$

We note that this is safe in the chiral limit $m_q,\, m_{q'} \to 0$, and for $m_q \to m_{q'}$ as well. For example, for $\langle \Lambda \bar{p} | \bar{s} u | 0 \rangle$ we have $(m_\Lambda - m_p)/(m_s - m_u) \sim 1$. For the modes studied here, the factor $(m_{\mathbf{B}} - m_{\mathbf{B}'})/(m_s - m_u)$ varies by 30, 40%, which illustrates SU(3) breaking.

The pseudoscalar and axial current matrix elements can be analogously related. Using $\langle \mathbf{B}\overline{\mathbf{B}}' | \partial^\mu A_\mu | 0 \rangle = (m_q + m_{q'}) \langle \mathbf{B}\overline{\mathbf{B}}' | \bar{q}\, \mathrm{i}\, \gamma_5 \, q' | 0 \rangle$, we have

$$g_{\mathrm{A}}(t) + \frac{t}{(m_{\mathbf{B}} + m_{\mathbf{B}'})^2} \, h_{\mathrm{A}}(t) = \frac{m_q + m_{q'}}{m_{\mathbf{B}} + m_{\mathbf{B}'}} \, g_{\mathrm{P}}(t). \quad (9)$$

As $m_q,\, m_{q'} \to 0$, we get $h_{\mathrm{A}}(t) \to -g_{\mathrm{A}}(t)\,(m_{\mathbf{B}} + m_{\mathbf{B}'})^2/t$ [9]. Since the $m_q/m_{\mathbf{B}}$ ratio is small, one is close to the chiral limit; hence the dependence of $h_{\mathrm{A}}(t)$ on $g_{\mathrm{P}}(t)$ is weak.

However, to ensure good chiral behavior, we previously followed [9] and took [10]

$$g_{\mathrm{P}}(t) = -g_{\mathrm{A}}(t) \, \frac{m^2_{\mathrm{GB}}(m_{\mathbf{B}} + m_{\mathbf{B}'})}{(m_q + m_{q'})(t - m^2_{\mathrm{GB}})}, \quad (10)$$

where m_{GB} is the corresponding Goldstone boson (e.g. kaon) mass. That is, $g_{\mathrm{P}}(t)$ is obtained by changing the $1/t$ term in the asymptotic form of $h_{\mathrm{A}}(t)$ to $1/(t - m^2_{\mathrm{GB}})$ and make use of (9) [9]. Indeed, (10) gave too small a rate for $B \to \Lambda \bar{p} \pi^+$ [9,10].

Due to the small quark–baryon mass ratio in (9), we note that g_{A} and h_{A} are insensitive to g_{P}. Therefore in the previous approach we need very precise information on both g_{A} and h_{A}, which is unavailable so far, to pinpoint g_{P}. In this work we choose a different strategy by studying $g_{\mathrm{A,P}}$ directly and obtaining h_{A} through (9).

According to QCD counting rules [12], both the vector form factor F_1 and the axial form factor g_{A}, supplemented by leading logs, behave as $1/t^2$ in the $t \to \infty$ limit. This is because we need two hard gluons to impart large momentum transfer. Similarly, considering the bilinear structure of the S and P operators, the scalar form factor f_{S} and pseudoscalar form factor g_{P} also behave as $1/t^2$ in the asymptotic limit. However, due to the need for a helicity flip, one needs an extra $1/t$ for F_2 and h_{A}; hence they behave as $1/t^3$.

We see that (10) gives a $1/t^3$ rather than a $1/t^2$ asymptotic behavior for g_{P}, which is symptomatic. In the electromagnetic current case, the asymptotic form has been confirmed by many experimental measurements of the nucleon (Sachs) magnetic form factor $G_{\mathrm{M}}^{p,n} = F_1^{p,n} + F_2^{p,n}$, over a wide range of momentum transfers in the space-like region. The asymptotic behavior for G_{M}^p also seems to hold in the time-like region, as reported by the Fermilab E760 experiment [13] for $8.9\,\mathrm{GeV}^2 < t < 13\,\mathrm{GeV}^2$. Another Fermilab experiment, E835, has recently reported [14] G_{M}^p for momentum transfers up to $\sim 14.4\,\mathrm{GeV}^2$. An empirical fit of $|G_{\mathrm{M}}^p| = Ct^{-2}[\ln(t/Q_0^2)]^{-2}$ is obtained, which is in agreement with the QCD counting rule.

The current induced form factors F_1, F_2 for the modes studied here can be related to the nucleon (Sachs) magnetic and electric form factors $G_{\mathrm{M,E}}$, as shown in Table 1, where we also give the SU(3) decomposition of g_{A} and g_{P} in terms of the form factors $D_{\mathrm{A,P}}$ and $F_{\mathrm{A,P}}$. The $F_1 + F_2$ terms are in fact obtained by using

$$D_V = -\frac{3}{2}\, G_{\mathrm{M}}^n, \qquad F_V = G_{\mathrm{M}}^p + \frac{1}{2}\, G_{\mathrm{M}}^n, \quad (11)$$

with SU(3) decompositions similar to that of $g_{\mathrm{A,P}}$. We can decompose f_{S} similarly into D_{S} and F_{S}, with (compare (8))

$$D_{\mathrm{S}} = \frac{m_{\mathbf{B}} - m_{\mathbf{B}'}}{m_q - m_{q'}} \left(-\frac{3}{2}\, F_1^n \right),$$

Table 1. Relations of baryon form factors $F_1 + F_2$, $g_{\rm A}$ and $g_{\rm P}$ with the nucleon magnetic form factors $G_{\rm M}^{p,n}$, $D_{\rm A,P}$ and $F_{\rm A,P}$ via the $(\bar{s}u)_{\rm V,A,P}$ operators. Replacing $G_{\rm M}^{p,n}$ by $G_{\rm E}^{p,n}$ in the second column, one obtains $F_1 + F_2\, t/(m_{\mathbf{B}} + m_{\overline{\mathbf{B}}'})^2$

$\mathbf{B}\overline{\mathbf{B}}'$	$F_1 + F_2$	$g_{\rm A,\,P}$
$\Lambda\,\bar{p}$	$-\sqrt{\frac{3}{2}}\,G_{\rm M}^p$	$-\frac{1}{\sqrt{6}}\,(D+3F)_{\rm A,P}$
$\Sigma^0\,\bar{p}$	$\frac{-1}{\sqrt{2}}\,(G_{\rm M}^p + 2\,G_{\rm M}^n)$	$\frac{1}{\sqrt{2}}\,(D-F)_{\rm A,P}$
$\Sigma^-\,\bar{n}$	$-(G_{\rm M}^p + 2\,G_{\rm M}^n)$	$(D-F)_{\rm A,P}$
$\Xi^0\,\overline{\Sigma}^+$	$G_{\rm M}^p - G_{\rm M}^n$	$(D+F)_{\rm A,P}$
$\Xi^-\,\overline{\Sigma}^0$	$\frac{1}{\sqrt{2}}\,(G_{\rm M}^p - G_{\rm M}^n)$	$\frac{1}{\sqrt{2}}\,(D+F)_{\rm A,P}$
$\Xi^-\,\overline{\Lambda}$	$\sqrt{\frac{3}{2}}\,(G_{\rm M}^p + G_{\rm M}^n)$	$-\frac{1}{\sqrt{6}}\,(D-3F)_{\rm A,P}$

$$F_{\rm S} = \frac{m_{\mathbf{B}} - m_{\mathbf{B}'}}{m_q - m_{q'}} \left(F_1^p + \frac{1}{2}\,F_1^n \right). \tag{12}$$

From the factorization assumption and Table 1, we expect

$$\begin{aligned} \mathcal{B}(\Lambda\,\bar{p}\,\pi^+) &\sim 2\,\mathcal{B}(\Lambda\,\bar{p}\,\pi^0), \\ \mathcal{B}(\Sigma^-\,\bar{n}\,\pi^+) &\sim 2\,\mathcal{B}(\Sigma^-\,\bar{n}\,\pi^0) \sim 2\,\mathcal{B}(\Sigma^0\,\bar{p}\,\pi^+) \\ &\sim 4\,\mathcal{B}(\Sigma^0\,\bar{p}\,\pi^0), \\ \mathcal{B}(\Xi^0\,\overline{\Sigma}^+\,\pi^+) &\sim 2\,\mathcal{B}(\Xi^0\,\overline{\Sigma}^+\,\pi^0) \sim 2\,\mathcal{B}(\Xi^-\,\overline{\Sigma}^0\,\pi^+) \\ &\sim 4\,\mathcal{B}(\Xi^-\,\overline{\Sigma}^0\,\pi^0). \end{aligned} \tag{13}$$

There are considerable data on the nucleon magnetic form factors. This allows us to make a fit [7]:

$$\begin{aligned} G_{\rm M}^p(t) &= \sum_{i=1}^{5} \frac{x_i}{t^{i+1}} \left[\ln\left(\frac{t}{\Lambda_0^2} \right) \right]^{-\gamma}, \\ G_{\rm M}^n(t) &= -\sum_{i=1}^{2} \frac{y_i}{t^{i+1}} \left[\ln\left(\frac{t}{\Lambda_0^2} \right) \right]^{-\gamma}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} \gamma &= 2.148\,, \\ x_1 &= 420.96\,\mathrm{GeV}^4\,, \\ x_2 &= -10485.50\,\mathrm{GeV}^6\,, \\ x_3 &= 106390.97\,\mathrm{GeV}^8\,, \\ x_4 &= -433916.61\,\mathrm{GeV}^{10}\,, \\ x_5 &= 613780.15\,\mathrm{GeV}^{12}\,, \\ y_1 &= 292.62\,\mathrm{GeV}^4\,, \\ y_2 &= -735.73\,\mathrm{GeV}^6\,, \end{aligned}$$

and $\Lambda_0 = 0.3\,\mathrm{GeV}$. They satisfy QCD counting rules and describe time-like electromagnetic data such as $e^+e^- \to N\overline{N}$ suitably well. The data are extracted by assuming $|G_{\rm E}^p| = |G_{\rm M}^p|$ and $|G_{\rm E}^n| = 0$ (which gives a better fit compared to the $|G_{\rm E}^n| = |G_{\rm M}^n|$ case [15]). With the fit of (14), the time-like $G_{\rm M}^{p(n)}$ is real and positive (negative) [16,17]. It is interesting to note that the fit coefficients x_i alternate in sign, and likewise for the y_i. Just two terms suffice for the latter because the neutron magnetic form factor data are relatively sparse [7]. According to perturbative QCD [18], asymptotically ($t \to \infty$) one expects $G_{\rm M}^n/G_{\rm M}^p = -2/3$. We find that the fitted parameters for $G_{\rm M}^n$ with the $|G_{\rm E}^n| = 0$ assumption give $G_{\rm M}^n/G_{\rm M}^p \to -y_1/x_1 = -0.70$, which is within 5% of the QCD expectation. Note that, by use of $G_{\rm M} = F_1 + F_2$ and asymptotically $F_2/F_1 \to 1/t \to 0$, we have $F_1^n/F_1^p \to G_{\rm M}^n/G_{\rm M}^p \to -2/3$ as well.

The F_2 term can be related to $(G_{\rm E} - G_{\rm M})/[t/(m_{\mathbf{B}} + m_{\mathbf{B}'})^2 - 1]$. However, we do not have many data on the time-like nucleon $G_{\rm E}$. Thus, we concentrate on the $F_1 + F_2$ term in (4) as we did in [7,10]. We also use $G_{\rm M}$ in place of F_1 in (8) and (12). The effect of the F_2 (or equivalently the $G_{\rm E} - G_{\rm M}$) contribution can be estimated by using form factor models such as vector meson dominance (VMD), where both $G_{\rm E}$ and $G_{\rm M}$ are available.

The time-like form factors related to $D_{\rm A}$, $F_{\rm A}$ are not yet measured, but, as pointed out in [9], their asymptotic behavior at $t \to \infty$ are known [19] and useful. Asymptotically, they can be described by two form factors, depending on the reacting quark having parallel or anti-parallel spin with respect to the baryon spin [19]. By expressing these two form factors in terms of $G_{\rm M}^{p,n}$ as $t \to \infty$, one has

$$D_{\rm A} \to G_{\rm M}^p + \frac{3}{2}\,G_{\rm M}^n, \qquad F_{\rm A} \to \frac{2}{3}\,G_{\rm M}^p - \frac{1}{2}\,G_{\rm M}^n. \tag{15}$$

In a similar fashion, in the asymptotic region the $f_{\rm S}$ and $g_{\rm P}$ form factors for the chirality flip operators S and P can be expressed by *just one* form factor, with the spin of the interacting quark parallel to the baryon spin. Anti-parallel spin corresponds to an octet–decuplet instead of an octet–octet baryon pair. Since $g_{\rm P}$ (equivalently $D_{\rm P}$, $F_{\rm P}$) and $f_{\rm S}$ are related to the *same* form factor, by following the approach of [19], as shown in Appendix A, we have

$$g_{\rm P} \to f_{\rm S}, \qquad \frac{D_{P(S)}}{F_{P(S)}} \to \frac{3}{2}, \tag{16}$$

as $t \to \infty$. This is a non-trivial requirement and it is not obeyed by (10). We note that (16) is obtained without the use of the equation of motion. The requirement of $D_{\rm S}/F_{\rm S} \to 3/2$ is consistent with (12), which follows from (8) by using $F_1^n/F_1^p \to G_{\rm M}^n/G_{\rm M}^p \to -2/3$ asymptotically [18]. Thus, the use of the equation of motion for $f_{\rm S}$ in (8) is consistent with the asymptotic relations in (16).

The asymptotic relations hold for large t, hence they imply relations on the leading terms of the corresponding form factors. In general, more terms would be needed. In analogy to the case of the neutron magnetic form factor, we express $D_{\rm A,P}$, $F_{\rm A,P}$ up to the second term [10],

$$\begin{aligned} D_{\rm A}(t) &\equiv \left(\frac{\tilde{d}_1}{t^2} + \frac{\tilde{d}_2}{t^3} \right) \left[\ln\left(\frac{t}{\Lambda_0^2} \right) \right]^{-\gamma}, \\ F_{\rm A}(t) &\equiv \left(\frac{\tilde{f}_1}{t^2} + \frac{\tilde{f}_2}{t^3} \right) \left[\ln\left(\frac{t}{\Lambda_0^2} \right) \right]^{-\gamma}, \\ D_{\rm P}(t) &\equiv \left(\frac{\bar{d}_1}{t^2} + \frac{\bar{d}_2}{t^3} \right) \left[\ln\left(\frac{t}{\Lambda_0^2} \right) \right]^{-\gamma}, \end{aligned}$$

$$F_{\rm P}(t) \equiv \left(\frac{\bar{f}_1}{t^2}+\frac{\bar{f}_2}{t^3}\right)\left[\ln\left(\frac{t}{\Lambda_0^2}\right)\right]^{-\gamma}. \quad (17)$$

The asymptotic relations of (15) and (16) imply $\tilde{d}_1 = x_1 - 3\,y_1/2$, $\tilde{f}_1 = 2\,x_1/3 + y_1/2$, $\bar{d}_1 = (3y_1/2)[(m_{\mathbf{B}} - m_{\mathbf{B}'})/(m_q - m_{q'})]$ and $\bar{f}_1 = (x_1 - y_1/2)[(m_{\mathbf{B}} - m_{\mathbf{B}'})/(m_q - m_{q'})]$, while further information is needed to determine $\tilde{d}_2$, $\tilde{f}_2$, $\bar{d}_2$ and $\bar{d}_2$, as we will discuss in the next section. We note that the anomalous dimensions of $g_{\rm P}$ and $f_{\rm S}$ may not be the same as that of $F_{1,2}$ and $g_{\rm A}$. However, their effect is logarithmic, and hence is not very important, and we apply the anomalous dimension of F_1 to the other ones for simplicity.

It is useful to compare with [10,9] for the treatment of $g_{\rm P}(t)$ (or equivalently on $h_{\rm A}(t)$), namely (10). As a working assumption, this form of $g_{\rm P}(t)$ with the $m^2_{\rm GB}/(m_q + m_{q'})$ factor was useful in particular for the good behavior of the pseudoscalar matrix element in the chiral limit. However, it may be too restrictive in three aspects: $g_{\rm P}(t) \propto g_{\rm A}(t)$, which is too strong an assumption; the appearance of the Goldstone boson pole in the time-like form factors, although $m_{\rm GB}$ is way below baryon pair threshold; and a $1/t^3$ asymptotic behavior, rather than the $1/t^2$ form as expected from QCD counting rules. Ultimately, it does not satisfy the asymptotic relation of (16). We have improved on these points in our present treatment of $g_{\rm P}(t)$.

3 Results

It is straightforward to use (2) to calculate $B \to \Lambda\,\bar{p}\,\pi$ and similar rates.

Before we start, let us first specify the parameters used. We take ϕ_3 (or γ) $= 60°$ [20] and the central values of $|V_{cb}|$ and $|V_{ub}|$ from [21]. We use $m_{u(d)}/m_s = 0.029\,(0.053)$, $m_s = 120\,\mathrm{MeV}$ and $m_b = 4.88\,\mathrm{GeV}$ at $\mu = 2.5\,\mathrm{GeV}$ [21, 22]. The $B \to \pi$ transition form factor is given in [23]. For the effective Wilson coefficients, we use $a_1 = 1.05$, $a_4 \times 10^4 = -387.3 - 121\mathrm{i}$ and $a_6 \times 10^4 = -555.3 - 121\mathrm{i}$ from [24] with $N_c = 3$.

Following [10], we use the axial vector contribution to $B^0 \to D^{*-}\,p\,\bar{n}$ decay to constrain $\tilde{f}_2$ and $\tilde{d}_2$. Since there is no scalar and pseudoscalar contribution in this tree dominated mode, we simply use the chiral limit form of $h_{\rm A}(t) = -g_{\rm A}(t)\,(m_p + m_n)^2/t$. The $g_{\rm P}$ contribution is suppressed by the quark–baryon mass ratio. We update our previous calculation [7] using the present input parameters, finding the vector part of the branching ratio to be $\mathcal{B}_V(D^{*-}\,p\,\bar{n}) = 11.9\,(a_1^{\rm eff}/0.85)^2 \times 10^{-4}$, where the same $a_1^{\rm eff}$ value as in [25] is used. To reach the central value of the measured rate $\mathcal{B}(B^0 \to D^{*-}\,p\,\bar{n}) = (14.5^{+3.4}_{-3.0} \pm 2.7) \times 10^{-4}$ [1], using $\tilde{d}_2 + \tilde{f}_2 = -956\,\mathrm{GeV}^6$ [1], we find $\mathcal{B}_{\rm A}(D^{*-}\,p\,\bar{n}) = 2.6\,(a_1^{\rm eff}/0.85)^2 \times 10^{-4}$ from the axial current. Although the value of $\tilde{d}_2 + \tilde{f}_2$ is about half of what was used in [10,25], the change only affects the branching ratios of the charmless modes studied here at the 10^{-8} level. Following [10], we use $\tilde{d}_2 = \tilde{f}_2$.

[1] By correcting a code error in [10], we can reproduce [25] the $\mathcal{B}_{\rm A}(D^*\,p\,\bar{n})$ result by using their $\tilde{d}_2 + \tilde{f}_2$ value determinded form the $D^0\,p\,\bar{p}$ rate. We do not use the $D^0\,p\,\bar{p}$ mode to fit the form factor parameters, since it is more complicated than the $D^*\,p\,\bar{n}$ mode

Table 2. Branching fractions for $\mathbf{B}\overline{\mathbf{B}}'\pi^+$ modes arising from the vector and scalar parts ($\mathcal{B}_{\rm V}$), and from the axial and pseudoscalar parts ($\mathcal{B}_{\rm A}$). The latter are given for the two cases of using the asymptotic $g_{\rm P}$ ($\bar{d}_2 = \bar{f}_2 = 0$) or the fitted $g_{\rm P}$ ($\bar{d}_2 = \bar{f}_2 = -952\ \mathrm{GeV}^6$) from the $\Lambda\,\bar{p}\,\pi$ rate. The branching fraction is a simple sum of the two, i.e. $\mathcal{B} = \mathcal{B}_{\rm V} + \mathcal{B}_{\rm A}$. The rates for the $\mathbf{B}\overline{\mathbf{B}}'\pi^0$ modes are about one half of those shown

Modes	$\mathcal{B}_{\rm V}(10^{-6})$	$\mathcal{B}_{\rm A}(10^{-6})$	
		use asymptotic $g_{\rm P}$	use fitted $g_{\rm P}$
$\Lambda\,\bar{p}\,\pi^+$	0.13	7.97	3.84
$\Sigma^0\,\bar{p}\,\pi^+$	0.88	0.70	0.70
$\Sigma^-\,\bar{n}\,\pi^+$	1.79	1.41	1.41
$\Xi^0\,\overline{\Sigma^+}\,\pi^+$	0.17	2.23	1.20
$\Xi^-\,\overline{\Sigma^0}\,\pi^+$	0.09	1.14	0.63
$\Xi^-\,\overline{\Lambda}\,\pi^+$	0.15	0.38	0.20

With the axial contribution fixed, and with the scalar and vector contribution related by the equation of motion (12) we give in Table 2 the vector plus scalar contribution ($\mathcal{B}_{\rm V}$) and the axial plus pseudoscalar contribution ($\mathcal{B}_{\rm A}$) to the $\overline{B}^0 \to \mathbf{B}\overline{\mathbf{B}}'\pi^+$ branching ratios. For $\mathcal{B}_{\rm A}$, we show two cases with either vanishing or non-vanishing $\bar{d}_2$ and $\bar{f}_2$ from the pseudoscalar form factor, which is yet to be fixed. Since the contribution from the vector plus scalar part does not interfere with the axial plus pseudoscalar part, the branching fraction is a simple sum of the two, i.e. $\mathcal{B} = \mathcal{B}_{\rm V} + \mathcal{B}_{\rm A}$, just as for $B^0 \to D^{*-}\,p\,\bar{n}$. By using the relation of (3), $\mathcal{B}(\mathbf{B}\overline{\mathbf{B}}'\pi^0)$ can be read off from Table 2 by a simple factor 1/2.

We find $\mathcal{B}_V(\Lambda[\Sigma^0]\,\bar{p}\,\pi^+) = 0.13\,[0.88] \times 10^{-6}$. We note that $\mathcal{B}_V(\Lambda\,\bar{p}\,\pi^+)$ is consistent with previous studies [10,9], while $\mathcal{B}_V(\Sigma^0\,\bar{p}\,\pi^+)$ becomes slightly larger because of the different input values of the neutron magnetic form factor parameters y_i. Clearly, the $\mathcal{B}_{\rm V}(\Lambda\,\bar{p}\,\pi^+)$ part is still an order of magnitude below the measured [11] branching ratio of (1). Before invoking the pseudoscalar form factor of (17), let us make sure that other modifications are insufficient for the order of magnitude difference.

Recall that in the vector and scalar sector, we concentrated on the $F_1 + F_2$ contributions without including the $G_{\rm E} - G_{\rm M}$ effect since $G_{\rm E}$ data are unavailable. As noted earlier, one can try to estimate the $G_{\rm E} - G_{\rm M}$ effect by using some form factor model where both $G_{\rm E}$ and $G_{\rm M}$ are given. We use a VMD model [16], which was discussed in our previous work [7]. Since $F_1^{\Lambda\bar{p}}(t)$ and $F_2^{\Lambda\bar{p}}(t)$ can be expressed in terms of $G^p_{\rm M}$ and $G^p_{\rm E}$, and since the VMD model describes the $G^p_{\rm M}$ data better than $G^n_{\rm M}$ (time-like) data [16], perhaps the $\Lambda\,\bar{p}\,\pi^+$ mode may be a better place to estimate the $G_{\rm E} - G_{\rm M}$ effect. By incorporating VMD with the previous section (following a similar approach as in [7]), we obtain $\mathcal{B}_{\rm V}(\Lambda\,\bar{p}\,\pi^+) = 0.27 \times 10^{-6}$. Although we gain by a factor of two compared to Table 2, the ef-

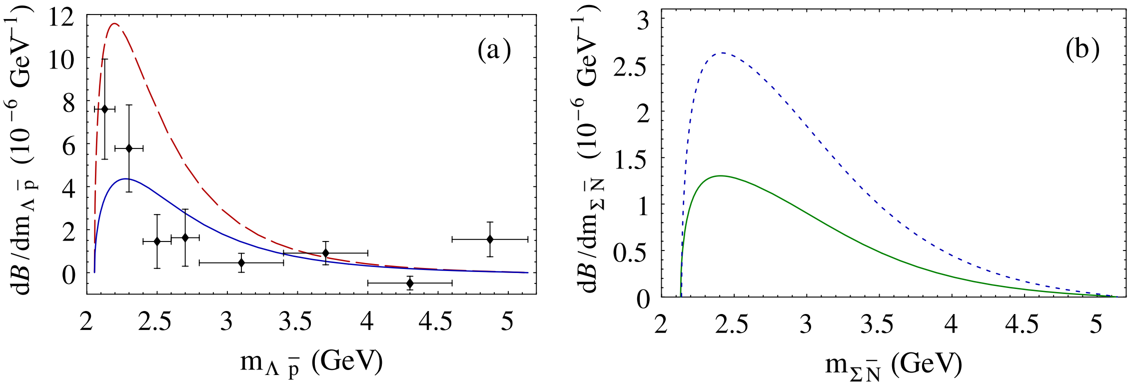

Fig. 1. **a** $\mathrm{d}\mathcal{B}(\Lambda \bar{p} \pi^+)/\mathrm{d}m_{\Lambda\bar{p}}$ spectrum, where a *solid* (*dashed*) line is for using the fitted (asymptotic) g_{P} of $\bar{d}_2 = \bar{f}_2 = -952\,\mathrm{GeV}^6$ (0); **b** $\mathrm{d}\mathcal{B}(\Sigma \bar{N} \pi^+)/\mathrm{d}m_{\Sigma\bar{N}}$ spectra, where the solid (dotted) line is for $\Sigma^0 \bar{p}$ ($\Sigma^- \bar{n}$). The plots for π^+ replaced by π^0 are expected to be similar but a factor of 2 lower

fect is still of order 10^{-7}, and is insufficient to account for the measured $\Lambda \bar{p} \pi^+$ rate. The effect of $G_{\mathrm{E}} - G_{\mathrm{M}}$ is not likely to fill the gap between $\mathcal{B}_{\mathrm{V}}(\Lambda \bar{p} \pi^+)$ and the measured $\mathcal{B}(\Lambda \bar{p} \pi^+)$.

We thus need to turn to the axial and pseudoscalar contributions. Let us start by using only the $\bar{d}_1$ and $\bar{f}_1$ terms of g_{P} determined by the asymptotic relation of (16), i.e. taking $\bar{d}_2 = \bar{f}_2 = 0$. It is remarkable that, as given in the first case for $\mathcal{B}_{\mathrm{A}}$ in Table 2 (column three), the $1/t^2$ terms of D_{P} and F_{P} alone give $\mathcal{B}(\Lambda \bar{p} \pi^+) \sim 8 \times 10^{-6}$, overshooting the experimental value by a factor of two! This is striking compared with the previous calculation using the ansatz of (10), which gave results an order of magnitude too small [10,9].

Now, we know that the sign of the x_i and y_i alternate; hence G_{M} gets reduced as higher power (in $1/t$) terms are included. We expect a similar effect for g_{P} by allowing for non-zero $\bar{d}_2$ and $\bar{f}_2$. We determine these coefficients (the $1/t^3$ terms) by fitting to the central value of the measured $\Lambda \bar{p} \pi^+$ rate. We obtain $-(\bar{d}_2 + 3\bar{f}_2)/\sqrt{6} = 1554.6\ \mathrm{GeV}^6$, which is displayed as the second case for $\mathcal{B}_{\mathrm{A}}$ in Table 2. By assuming $\bar{d}_2 \sim \bar{f}_2$, we have $\bar{d}_2 \sim -952\ \mathrm{GeV}^6$, which has a sign opposite to $\bar{d}_1$, and is about twice the size of $\tilde{d}_2 = \tilde{f}_2 = -478\ \mathrm{GeV}^6$, the $1/t^3$ coefficients for the axial vector form factor.

We show in Fig. 1a the $\Lambda \bar{p} \pi^+$ decay spectrum. It is interesting that the predicted spectra in both the $\bar{d}_2 = \bar{f}_2 = 0$ and $\bar{d}_2 = \bar{f}_2 = -952\ \mathrm{GeV}^6$ cases are close to the data. The data suggest a curve between these two, which conforms with our expectation that the third, $1/t^4$, term would have the same sign as the $1/t^2$ term. In the future as the measured spectrum is improved, one may in turn use it to extract baryon time-like form factors.

While $\mathcal{B}(\Lambda \bar{p} \pi^+)$ is enhanced from the previous results [10,9] by using our new approach to the pseudoscalar g_{P} form factor, the enhancement in $\mathcal{B}(\Sigma^0 \bar{p} \pi^+)$ turns out to be rather mild. This can be understood from the relative weight of Λ versus Σ^0 in (A.5) of Appendix A. We expect $\mathcal{B}(\Sigma^0 \bar{p} \pi^+) = 1.6 \times 10^{-6}$, which is within the present Belle limit of $\mathcal{B}(\Sigma^0 \bar{p} \pi^+) < 3.8 \times 10^{-6}$ at 90% confidence level [11]. Furthermore, the SU(3) predictions of $\mathcal{B}(\Sigma^- \bar{n} \pi^+) \sim 2\,\mathcal{B}(\Sigma^0 \bar{p} \pi^+)$ and $\mathcal{B}(\Xi^0 \overline{\Sigma^+} \pi^+) \sim 2\,\mathcal{B}(\Xi^- \overline{\Sigma^0} \pi^+)$ given in Table 2 are easy to verify.

In Fig. 1b we plot the $\Sigma^0 \bar{p} \pi^+$ and $\Sigma^- \bar{n} \pi^+$ decay spectra. The $\Sigma^0 \bar{p} \pi^+$ spectrum is close to our previous calculation in [10]. Since the corresponding SU(3) decomposition for these two modes is $D_{\mathrm{P}} - F_{\mathrm{P}}$, the rates are not sensitive to $\bar{d}_2$ and $\bar{f}_2$ being zero or finite, so long as they are not too different from each other.

We show in Fig. 2 the $\Xi^0 \overline{\Sigma^+} \pi^+$, $\Xi^- \overline{\Sigma^0} \pi^+$ and $\Xi^- \overline{\Lambda} \pi^+$ decay spectra with $\bar{d}_2$ and $\bar{f}_2$ zero or finite.

We expect Figs. 1 and 2 to give also the spectra of the modes with π^+ replaced by π^0, but with a factor of two reduction in the rate from the isospin factor.

In these three-body modes quite often we have a Λ hyperon produced, which is well known to self-analyze its spin upon decay and to provide useful information for possible CP- and T-violation and chirality studies in B decays [6,26]. Following [26], the angular distribution of the cascade $\overline{B} \to \Lambda \bar{p} \pi \to \pi^- p \bar{p} \pi$ decay can be written as

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_\Lambda \mathrm{d}\cos\theta} = \frac{1}{2}\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\Lambda}[1 + \overline{\alpha}_\Lambda(E_\Lambda)\cos\theta], \qquad (18)$$

where E_Λ is the Λ energy measured in the $\overline{B}$ rest frame and θ is the supplementary angle between the emitted proton momentum and the $\overline{B}$ momentum in the Λ rest frame. We have $\overline{\alpha}_\Lambda(E_\Lambda) = \mathcal{P}_\Lambda(E_\Lambda)\,\alpha_\Lambda$, where the Λ polarization $\mathcal{P}_\Lambda(E_\Lambda)$ is given in Appendix B and $\alpha_\Lambda = 0.642 \pm 0.013$ [21] is the well-measured Λ decay asymmetry parameter.

We show in Fig. 3 the asymmetry $\overline{\alpha}_\Lambda(E_\Lambda)$ and the $\mathrm{d}\mathcal{B}(\Lambda \bar{p} \pi^+)/\mathrm{d}E_\Lambda$ spectrum. The $\overline{\alpha}_\Lambda(E_\Lambda)$ plot is similar to the plot shown in [26] obtained by using some general arguments. The negative $\overline{\alpha}_\Lambda(E_\Lambda)$ corresponds to a left-handed helicity dominated Λ in B decay. It is interesting to note that although the decay rate is dominated by the pseudoscalar term, we still have a polarized Λ. This can be understood by noting that the ratio of scalar and pseudoscalar contributions is roughly given by the averaged $f_{\mathrm{S}}^2/g_{\mathrm{P}}^2$, which is about 0.1, while the polarization $\mathcal{P}_\Lambda$ is roughly given by the averaged $-2f_{\mathrm{S}}\,g_{\mathrm{P}}/(f_{\mathrm{S}}^2 + g_{\mathrm{P}}^2) \sim -2f_{\mathrm{S}}/g_{\mathrm{P}}$, which can be as large as -0.6. The sharp turn of $\overline{\alpha}_\Lambda(E_\Lambda)$ towards a much more negative value for $E_\Lambda >$

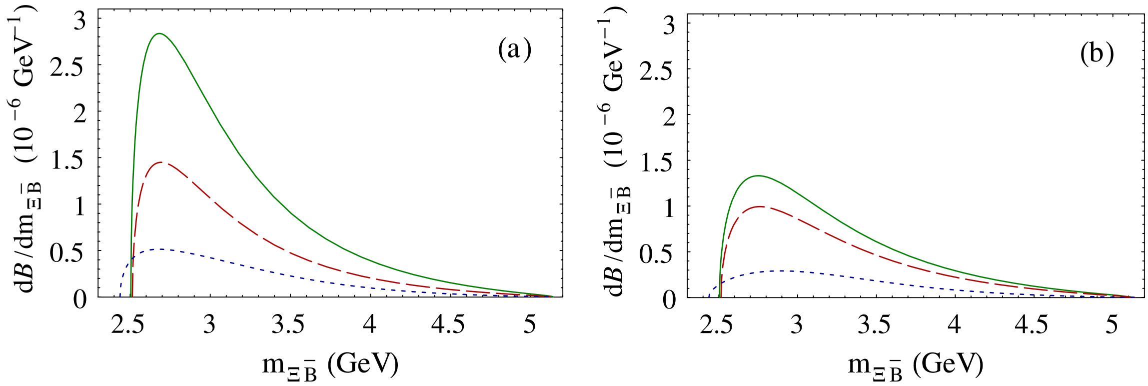

Fig. 2a,b. *Solid*, *dashed* and *dotted lines* are for $\mathrm{d}\mathcal{B}(\Xi^0\,\overline{\Sigma^+}\,\pi^+)/\mathrm{d}m_{\Xi^0\,\overline{\Sigma^+}}$, $\mathrm{d}\mathcal{B}(\Xi^-\,\overline{\Sigma^0}\,\pi^+)/\mathrm{d}m_{\Xi^-\,\overline{\Sigma^0}}$ and $\mathrm{d}\mathcal{B}(\Xi^-\,\overline{\Lambda}\,\pi^+)/\mathrm{d}m_{\Xi^-\,\overline{\Lambda}}$, respectively, for using **a** the asymptotic g_P ($\bar{d}_2 = \bar{f}_2 = 0$), and **b** the fitted g_P ($\bar{d}_2 = \bar{f}_2 = -952\,\mathrm{GeV}^6$)

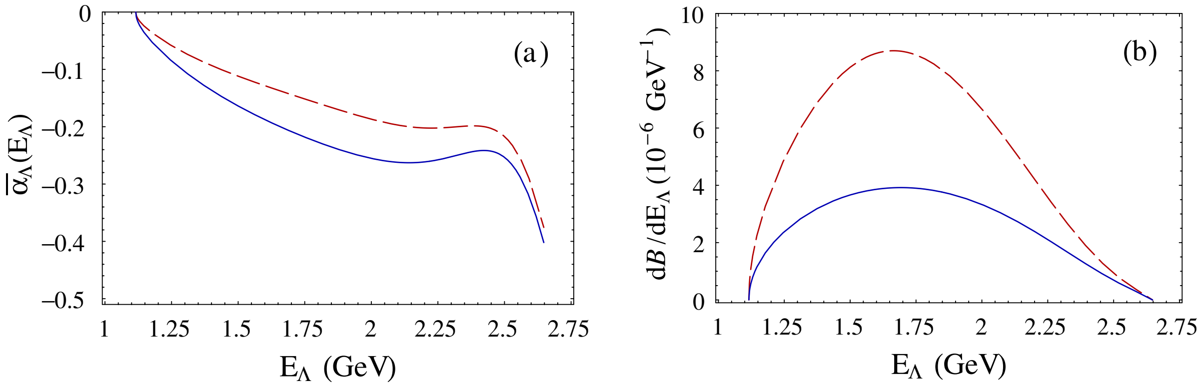

Fig. 3. **a** $\overline{\alpha}_\Lambda(E_\Lambda)$, **b** $\mathrm{d}\mathcal{B}(\Lambda\,\bar{p}\,\pi^+)/\mathrm{d}E_\Lambda$ spectrum, where the *solid* (*dashed*) line is for using the fitted (asymptotic) g_P of $\bar{d}_2 = \bar{f}_2 = -952\,\mathrm{GeV}^6$ (0)

2.5 GeV is due to the fact that as E_Λ increases, the phase space quickly reduces to a high $m_{\Lambda\bar{p}}$ region, resulting in the approach of g_P to f_S and consequently the increase in left-handed Λ polarization. It is well known that the Λ spin is mainly carried by the s quark (as shown in (A.2)) and it is left-handed in the $\overline{B} \to \Lambda\,\bar{p}\,\pi$ decay (as shown in (2)). Therefore, a dominantly left-handed Λ reflects the V–A nature of the weak interaction [26]. By comparing Fig. 3a and b, we find $-\overline{\alpha}_\Lambda \sim 0.2$–$0.3$ for the main portion of $\Lambda\,\bar{p}\,\pi$ events. One should be able to check the sign of this asymmetry experimentally in the near future.

4 Discussion and conclusion

Let us check the ϕ_3 dependence of the modes considered here. For all modes, $\mathcal{B}_\mathrm{V}$ increases as we change ϕ_3 from 60° to 90°; on the other hand, $\mathcal{B}_\mathrm{A}$ increases for $\Lambda\,\bar{p}\,\pi$, $\Sigma\,\overline{N}\,\pi$ and $\Xi^-\,\overline{\Lambda}\,\pi$, but decreases for $\Xi^0\,\overline{\Sigma}^+\,\pi$ and $\Xi^-\,\overline{\Sigma}^0\,\pi$. However, the variations are at order 10^{-7} and far less significant compared to the $K\pi$ case [27]. Since the $a_6\,V_{tb}\,V^*_{ts}$ terms dominate, we do not expect a strong dependence on ϕ_3 or N_c. Similarly, single term dominance implies that direct CP-violation cannot be large; this is found to be within $+5\%$ for all modes.

It is interesting to discuss the implication on $p\,\bar{p}\,K$ and $p\,\bar{p}\,\pi$ modes calculated in [9,10]. First of all, the changes are in the current-produced parts, whereas these modes contain transition parts as well. In particular, the $p\,\bar{p}\,\pi^-$ mode is transition dominated. From (9), we see that $h_\mathrm{A}(t)$ is close to its chiral limit form because the dependence on g_P is rather weak, and $h_\mathrm{A}(t)$ for the present work is similar to previous [9,10]. Therefore, the axial vector contributions to the $p\,\bar{p}\,K$ and $p\,\bar{p}\,\pi$ modes are not affected. The effect of g_P only enters through the pseudoscalar term. Since the pseudoscalar matrix element for $B \to p\,\bar{p}\,K$ decay, $\langle p\bar{p}\,|(\bar{s}s)_\mathrm{P}|0\rangle$ [10], is Okubo–Zweig–Iizuka (OZI) suppressed, we do not expect much change in these modes. On the other hand, for $\langle p\bar{p}\,|(\bar{d}d)_\mathrm{P}|0\rangle$ of the $p\,\bar{p}\,\pi^-$ mode, by using SU(3) and OZI arguments as in [10], it corresponds to $F_\mathrm{P} - D_\mathrm{P}$ and is non-negligible. However, this mode is tree and transition dominant; hence we still do not expect much change in the rate [10]. Note that the transition form factor has a $1/t^3$ behavior. For large enough t, the transition part is power suppressed. We thus expect to see some $1/t^2$ contribution from the new g_P term, resulting in a slightly broader spectrum than previous [10].

In conclusion, we study decay rates and spectra of $\overline{B} \to \Lambda\,\bar{p}\,\pi$, $\Sigma^0\,\bar{p}\,\pi$, $\Sigma^-\,\bar{n}\,\pi$, $\Xi^0\,\overline{\Sigma}^+\,\pi$, $\Xi^-\,\overline{\Sigma}^0\,\pi$ and $\Xi^-\,\overline{\Lambda}\,\pi$ modes, and the Λ polarization in this work. By suitably incorporating the asymptotic behavior of the baryonic pseudoscalar matrix element, we are able to obtain the $\Lambda\,\bar{p}\,\pi^+$ rate (in part by a fit) and spectrum close to the experimental measurements. The discrepancy between experimental [11] and previous theoretical [9,10] results is perhaps resolved. While the $\Lambda\,\bar{p}\,\pi^+$ rate is enhanced from the previous calculation, we expect $\mathcal{B}(\Sigma^0\,\bar{p}\,\pi^+) = 1.6 \times 10^{-6}$, which is within the present experimental limit and can be checked soon. Although the $\Lambda\,\bar{p}\,\pi^+$ rate is dominated by the pseudoscalar term, we still have Λ polarized giving $\overline{\alpha}_\Lambda \sim -(0.2\text{–}0.3)$. The impact on $p\,\bar{p}\,K$ due to the present treatment of the pseudoscalar form factor is negligible, while we expect a slight broadening of the $p\,\bar{p}\,\pi^-$ spectrum. Most of the subtleties in these modes come from the axial and especially the pseudoscalar form factors. Information on these form factors may be obtained from studying these modes. However, the underlying factorization assumption needs to be checked separately. It is interesting that factorization seems to work in the $\overline{B}^0 \to D^+K^-K^0$ and $D^{*+}K^-K^0$ modes, where axial parts are absent and vector parts are known [28]. For these current-produced three-body baryonic modes, we expect $\mathcal{B}(\mathbf{B}\overline{\mathbf{B}}'\pi^+) \sim 2\,\mathcal{B}(\mathbf{B}\overline{\mathbf{B}}'\pi^0)$ as a consequence of factorization, which does not depend on the complexity of the baryonic form factors.

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Appendix

A Asymptotic relations for the form factors f_S and g_P

We follow [19] to obtain the asymptotic relations for f_S and g_P. The wave function of an octet baryon can be expressed as

$$|\mathbf{B}\,;\uparrow\rangle \sim \frac{1}{\sqrt{3}}(|\mathbf{B}\,;\uparrow\downarrow\uparrow\rangle + |\mathbf{B}\,;\uparrow\uparrow\downarrow\rangle + |\mathbf{B}\,;\downarrow\uparrow\uparrow\rangle), \quad \text{(A.1)}$$

i.e. composed of 13-, 12- and 23-symmetric terms, respectively. For $\mathbf{B} = p$, n, Σ^0, Λ, we have

$$\begin{aligned}
|p\,;\uparrow\downarrow\uparrow\rangle &= \left[\frac{d(1)u(3)+u(1)d(3)}{\sqrt{6}}u(2) - \sqrt{\frac{2}{3}}u(1)d(2)u(3)\right]|\uparrow\downarrow\uparrow\rangle,\\
|n\,;\uparrow\downarrow\uparrow\rangle &= (-|p\,;\uparrow\downarrow\uparrow\rangle \text{ with } u \leftrightarrow d),\\
|\Sigma^0\,;\uparrow\downarrow\uparrow\rangle &= \left[-\frac{u(1)d(3)+d(1)u(3)}{\sqrt{3}}\,s(2) + \frac{u(2)d(3)+d(2)u(3)}{2\sqrt{3}}\,s(1) + \frac{u(1)d(2)+d(1)u(2)}{2\sqrt{3}}\,s(3)\right]|\uparrow\downarrow\uparrow\rangle,\\
|\Lambda\,;\uparrow\downarrow\uparrow\rangle &= \left[\frac{d(2)u(3)-u(2)d(3)}{2}\,s(1) + \frac{u(1)d(2)-d(1)u(2)}{2}\,s(3)\right]|\uparrow\downarrow\uparrow\rangle, \quad \text{(A.2)}
\end{aligned}$$

for the corresponding $|\mathbf{B}\,;\uparrow\downarrow\uparrow\rangle$ parts, while the 12- and 23-symmetric parts can be obtained by permutation. To be consistent with the SU(3) decompositions of Table 1, our Λ state has an overall negative sign with respect to that of [19].

Following [19], we have

$$\begin{aligned}
\langle\mathbf{B}(p)|\mathcal{O}|\mathbf{B}'(p')\rangle &= \bar{u}(p)\left[\frac{1+\gamma_5}{2}\,F^+(t) + \frac{1-\gamma_5}{2}\,F^-(t)\right]u(p'),\\
F^\pm(t) &= e_\parallel^{(\pm)}(\mathcal{O}:\mathbf{B}'\to\mathbf{B})\,F_\parallel(t), \quad \text{(A.3)}
\end{aligned}$$

in the large t limit. Quark mass dependent terms behave like $m_q/\sqrt{|t|}$ and are neglected. For simplicity, we illustrate the derivation of the asymptotic relations with the space-like case.

The coefficients of $F_\parallel$ for the $\mathcal{O} = \bar{q}_\mathrm{L}q'_\mathrm{R}$, $\bar{q}_\mathrm{L}q'_\mathrm{R}$ cases are given by

$$\begin{aligned}
&e_\parallel^+(\bar{q}_\mathrm{L}q'_\mathrm{R}:\mathbf{B}'\to\mathbf{B})\\
&= \langle\mathbf{B};\downarrow\downarrow\uparrow|Q[q'(1,\uparrow)\to q(1,\downarrow)]|\mathbf{B}'\,;\uparrow\downarrow\uparrow\rangle\\
&\quad+\langle\mathbf{B};\uparrow\downarrow\downarrow|Q[q'(3,\uparrow)\to q(3,\downarrow)]|\mathbf{B}'\,;\uparrow\downarrow\uparrow\rangle,\\
&e_\parallel^-(\bar{q}_\mathrm{L}q'_\mathrm{R}:\mathbf{B}'\to\mathbf{B}) = 0,\\
&e_\parallel^\pm(\bar{q}_\mathrm{R}q'_\mathrm{L}:\mathbf{B}'\to\mathbf{B}) = e_\parallel^\mp(\bar{q}_\mathrm{L}q'_\mathrm{R}:\mathbf{B}'\to\mathbf{B}), \quad \text{(A.4)}
\end{aligned}$$

where $Q[q'(1(3),\uparrow)\to q(1(3),\downarrow)]$ change the parallel spin $q'(1(3))|\uparrow\rangle$ part of $|\mathbf{B}';\uparrow\downarrow\uparrow\rangle$ to a $q(1(3))|\downarrow\rangle$ part. It is easy to see that flipping the anti-parallel spin $|\downarrow\rangle$ part of $|\mathbf{B}';\uparrow\downarrow\uparrow\rangle$ to $|\uparrow\rangle$ will give a decuplet instead of an octet state. Thus, we need to consider the parallel spin case only. By using the above equations, it is straightforward to obtain

$$\begin{aligned}
e_\parallel^+(\bar{u}_\mathrm{L}d_\mathrm{R}:n\to p) &= -\frac{5}{3},\\
e_\parallel^+(\bar{u}_\mathrm{L}s_\mathrm{R}:\Lambda\to p) &= \sqrt{\frac{3}{2}},\\
e_\parallel^+(\bar{u}_\mathrm{L}s_\mathrm{R}:\Sigma^0\to p) &= -\frac{1}{3\sqrt{2}}. \quad \text{(A.5)}
\end{aligned}$$

By using $S, P = \bar{q}_\mathrm{L}q'_\mathrm{R} \pm \bar{q}_\mathrm{R}q'_\mathrm{L}$ and (A.4), we have $e_\parallel^\pm(\bar{q}q':\mathbf{B}'\to\mathbf{B}) = e_\parallel^+(\bar{q}_\mathrm{L}q'_\mathrm{R}:\mathbf{B}'\to\mathbf{B})$ and $e_\parallel^\pm(\bar{q}\gamma_5q':\mathbf{B}'\to\mathbf{B}) = \pm e_\parallel^+(\bar{q}_\mathrm{L}q'_\mathrm{R}:\mathbf{B}'\to\mathbf{B})$. Hence

$$f_\mathrm{S} = g_\mathrm{P} = e_\parallel^+(\bar{q}_\mathrm{L}q'_\mathrm{R}:\mathbf{B}'\to\mathbf{B})\,F_\parallel, \quad \text{(A.6)}$$

in the large t limit. In terms of $D_{\mathrm{S(P)}}$ and $F_{\mathrm{S(P)}}$, we have $f_\mathrm{S}(g_\mathrm{P}) = D_{\mathrm{S(P)}} + F_{\mathrm{S(P)}}$, $-(D_{\mathrm{S(P)}} + 3F_{\mathrm{S(P)}})/\sqrt{6}$, $(D_{\mathrm{S(P)}} -$

$F_{\mathrm{S(P)}})/\sqrt{2}$ for the $\mathbf{B'B} = np$, Λp and $\Sigma^0 p$ cases, respectively. Accordingly,

$$D_{\mathrm{S}} = D_{\mathrm{P}} = -F_{\parallel}\,, \qquad F_{\mathrm{S}} = F_{\mathrm{P}} = -\frac{2}{3}\,F_{\parallel}\,, \tag{A.7}$$

which implies (16).

B Decay rate and polarization formula

For a three-body $B \to h\mathbf{B}\overline{\mathbf{B}}'$ decay, where h is a pseudoscalar meson and $\mathbf{B}$, $\overline{\mathbf{B}}'$ is a baryon–anti-baryon pair, in general the amplitude can be written as

$$\begin{aligned}&\mathcal{M}\left(B \to h\mathbf{B}\overline{\mathbf{B}}'\right)\\ &= \frac{G_{\mathrm{F}}}{\sqrt{2}}\Big\{\mathcal{A}\,\bar{u}(p_{\mathbf{B}})\,\not{p}_h v(p_{\overline{\mathbf{B}}'}) + \mathcal{B}\,\bar{u}(p_{\mathbf{B}})\,\not{p}_h\gamma_5 v(p_{\overline{\mathbf{B}}'})\\ &\quad + \mathcal{C}\,\bar{u}(p_{\mathbf{B}})v(p_{\overline{\mathbf{B}}'}) + \mathcal{D}\,\bar{u}(p_{\mathbf{B}})\gamma_5 v(p_{\overline{\mathbf{B}}'})\Big\}.\end{aligned} \tag{B.1}$$

The decay rate is given by

$$\mathrm{d}\Gamma = \frac{1}{(2\,\pi)^3}\,\frac{1}{32\,m_B^3}\left(\Sigma_{\lambda_{1,2}}\,|\mathcal{M}|^2\right)\mathrm{d}m_{12}^2\,\mathrm{d}m_{23}^2\,, \tag{B.2}$$

where we call the baryon $\mathbf{B}$ particle 1, the anti-baryon $\overline{\mathbf{B}}'$ particle 2 and the meson h particle 3, and $\lambda_{1(2)} = \pm 1$ is the helicity of the (anti-) baryon $\mathbf{B}$ ($\overline{\mathbf{B}}'$).

If the baryon $\mathbf{B}$ is in a definite helicity state, its spin direction will remain the same in either the B meson or its own rest frames. For the baryon $\mathbf{B}$ with energy E_1 (measured in the B meson rest frame) the density matrix in the spin (or helicity) space is given by

$$\rho(E_1) = \frac{1}{2}\left[1 + \mathcal{P}_{\mathbf{B}}(E_1)\,\hat{p}_1\cdot\sigma\right], \tag{B.3}$$

where $\hat{p}_1$ is the unit vector pointing opposite to the direction of the B meson momentum in the $\mathbf{B}$ baryon rest frame and

$$\mathcal{P}_{\mathbf{B}}(E_1) = \frac{\int \mathrm{d}m_{23}^2\,\Sigma_{\lambda_{1,2}}(-)^{\lambda_1}|\mathcal{M}|^2}{\int \mathrm{d}m_{23}^2\,\Sigma_{\lambda_{1,2}}|\mathcal{M}|^2}. \tag{B.4}$$

It is straightforward to obtain

$$\begin{aligned}&\Sigma_{\lambda_{1,2}}\,(-)^{\lambda_1}|\mathcal{M}|^2\\ &= G_{\mathrm{F}}^2\,4\Big\{\mathrm{Re}(\mathcal{A}\,\mathcal{B}^*)m_1(2s_1\cdot p_3\,p_2\cdot p_3 - m_3^2\,s_1\cdot p_2)\\ &\quad + \mathrm{Re}\,(\mathcal{A}\,\mathcal{D}^* - \mathcal{B}\,\mathcal{C}^*)\,m_1 m_2\,s_1\cdot p_3\\ &\quad + \mathrm{Re}\,(\mathcal{A}\,\mathcal{D}^* + \mathcal{B}\,\mathcal{C}^*)\,(s_1\cdot p_3\,p_1\cdot p_2 - s_1\cdot p_2\,p_1\cdot p_3)\\ &\quad - \mathrm{Re}\,(\mathcal{C}\,\mathcal{D}^*)\,m_1\,s_1\cdot p_2\Big\},\end{aligned} \tag{B.5}$$

$$\begin{aligned}&\Sigma_{\lambda_{1,2}}\,|\mathcal{M}|^2\\ &= G_{\mathrm{F}}^2\,2\Big\{\Big[|\mathcal{A}|^2(2p_1\cdot p_3\,p_2\cdot p_3 - m_3^2\,p_1\cdot p_2 - m_1 m_2 m_3^2)\\ &\quad + 2\,\mathrm{Re}\,(\mathcal{A}\,\mathcal{C}^*)\,(m_1 p_2\cdot p_3 - m_2\,p_1\cdot p_3)\\ &\quad + |\mathcal{C}|^2(p_1\cdot p_2 - m_1 m_2)\Big]\\ &\quad + [\mathcal{A}\to\mathcal{B},\,\mathcal{C}\to\mathcal{D},\,m_2\to -m_2]\Big\},\end{aligned} \tag{B.6}$$

where s_1 is the helicity vector of the baryon $\mathbf{B}$ (spinor) with $\lambda_1 = +1$. It is easy to check that by neglecting m_1 we have $m_1 s_1 \to p_1$ and we obtain $\mathcal{P}_{\mathbf{B}}(E_1) \to -1$ in the fully left-handed chiral case ($\mathcal{A} \sim -\mathcal{B}$ and $\mathcal{C} \sim \mathcal{D}$) as expected from (B.1). In general, the polarization $\mathcal{P}_{\mathbf{B}}(E_1)$ can easily be evaluated in the B meson rest frame by using

$$s_1 = \frac{1}{m_1\sqrt{(p_B\cdot p_1)^2 - m_1^2 m_B^2}}(p_B\cdot p_1\,p_1 - m_1^2\,p_B), \tag{B.7}$$

where p_B is the momentum of the B meson, and the standard technique of expressing $p_B\cdot p_i$, $p_i\cdot p_j$ in terms of m_{ij}^2. Given these formulas, the task is now reduced to extract the $\mathcal{A}$–$\mathcal{D}$ terms for the amplitude of interest.

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